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THE WIRELESS NETWORK JAMMING PROBLEM (PREPRINT)

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In adversarial environments, disabling the communication capabilities of the enemy is a high priority. We introduce the problem of determining the optimal number and locations for a set of jamming devices in order to neutralize a wireless communication network. This problem is known as the WIRELESS NETWORK JAMMING PROBLEM. We develop several mathematical programming formulations based on covering the communication nodes and limiting the connectivity index of the nodes. Two case studies are presented comparing the formulations with the addition of various percentile constraints. Finally, directions of further research are addressed.

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ABSTRACT. In adversarial environments, disabling the communication capabilities of the enemy is a high priority. We introduce the problem of determining the optimal number and locations for a set of jamming devices in order to neutralize a wireless communication network. This problem is known as the WIRELESS NETWORK JAMMING PROBLEM. We develop several mathematical programming formulations based on covering the communication nodes and limiting the connectivity index of the nodes. Two case studies are presented comparing the formulations with the addition of various percentile constraints. Finally, directions of further research are addressed.

1. INTRODUCTION

Military strategists are constantly seeking ways to increase the effectiveness of their force while reducing the risk of casualties. In any adversarial environment, an important goal is always to neutralize the communication system of the enemy. In this work, we are interested in jamming a wireless communication network. Specifically, we introduce and study the problem of determining the optimal number and placement for a set of jamming devices in order to neutralize communication on the network. This is known as the WIRE-LESS NETWORK JAMMING PROBLEM (WNJP). Despite the enormous amount of research on optimization in telecommunications [6], this important problem for military analysts has received little attention by the research community.

The organization of the paper is as follows. Section 2 contains several formulations based on covering the communication nodes with jamming devices. In Section 3, we use tools from graph theory to define an alternative formulation based on limiting the connectivity index of the network nodes. Next, we incorporate percentile constraints to develop formulations which provide solutions requiring less jamming devices, but whose solution quality favors the exact methods. In Section 5, we present two case studies comparing the solutions and computation time for all formulations. Finally, conclusions and future directions of research are addressed.

2. COVERAGE FORMULATIONS

Before formally defining the problem statement, we will state some basic assumptions about the jamming devices and the communication nodes being jammed. We assume that the such parameters as the frequency range of the jamming devices are known. In addition, the jamming devices are assumed to have omnidirectional antennas. The communication nodes are also assumed to be outfitted with omnidirectional antennas and function as both receivers and transmitters. Given a graph $G = (V, E)$, we can represent the communication devices as the vertices of the graph. An undirected edge would connect two nodes if they are within a certain communication threshold.

Given a set $\mathcal{M} = \{1, 2, \ldots, m\}$ of communication nodes to be jammed, the goal is to find a set of locations for placing jamming devices in order to suppress the functionality of the network. The *jamming effectiveness* of device j is calculated using $d : (V \times V) \to \mathbb{R}$, where d is a decreasing function of the distance from the jamming device to the node being

jammed. Here we are considering radio transmitting nodes, and correspondingly, jamming devices which emit electromagnetic waves. Thus the jamming effectiveness of a device depends on the power of its electromagnetic emission, which is inversely proportional to the squared distance from the jamming device to the node being jammed. Specifically,

$$d_{ij} = \frac{\lambda}{r^2(i,j)},$$

where $\lambda \in \mathbb{R}$ is a constant, and $r(i,j)$ represents the distance between node i and jamming device j. Without the loss of generality, we can set $\lambda = 1$.

The cumulative level of jamming energy received at node i is defined as

$$Q_i = \sum_{j=1}^{n} d_{ij} = \sum_{j=1}^{n} \frac{1}{r^2(i,j)},$$

where n is the number of jamming devices. Then, we can formulate the WIRELESS NETWORK JAMMING PROBLEM (WNJP) as the minimization of the number of jamming devices placed, subject to a set of *quality covering* constraints:

$$\textbf{(QCP)} \quad \text{Minimize} \quad n \tag{1}$$
$$\text{s.t.} \quad Q_i \geq C_i, \quad i = 1, 2, \ldots, m. \tag{2}$$

The solution to this problem provides the optimal number of jamming devices needed to ensure a certain jamming threshold C_i is met at every node $i \in \mathcal{M}$. A continuous optimization approach where one is seeking the optimal placement coordinates $(x_j, y_j), j = 1, 2, \ldots, n$ for jamming devices given the coordinates $(X_i, Y_i), i = 1, 2, \ldots, m$, of network nodes, leads to highly non-convex formulations. For example, consider the quality covering constraint for network node i,

$$\sum_{j=1}^{n} \frac{1}{(x_j - X_i)^2 + (y_j - Y_i)^2} \geq C_i.$$

It is easy to verify that this constraint is non-convex. Finding the optimal solution to this nonlinear programming problem would require an extensive amount of computational effort.

To overcome the non-convexity of the above formulation, we propose several integer programming models for the problem. Suppose now that along with the set of communication nodes $\mathcal{M} = \{1, 2, \ldots, m\}$, there is a fixed set $\mathcal{N} = \{1, 2, \ldots, n\}$ of possible locations for the jamming devices. This assumption is reasonable because in real battlefield scenarios, the set of possible placement locations will likely be limited. Define the decision variable x_j as

$$x_j = \begin{cases} 1, & \text{if a jamming device is installed at location } j \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

If we redefine $r(i,j)$ to be the distance between communication node i and jamming location j, then we have the OPTIMAL NETWORK COVERING (ONC) formulation of the WNJP as

$$\textbf{(ONC)} \quad \text{Minimize} \quad \sum_{j=1}^{n} c_j x_j \tag{4}$$
$$\text{s.t.}$$
$$\sum_{j=1}^{n} d_{ij} x_j \geq C_i, \quad i = 1, 2, \ldots, m \tag{5}$$
$$x_j \in \{0, 1\}, \quad j = 1, 2, \ldots, n, \tag{6}$$

where C_i is defined as above. Here the objective is to minimize the number of jamming devices used while achieving some minimum level of coverage at each node. The coefficients c_j in (4) represent the costs of installing a jamming device at location j. In a battlefield scenario, placing a jamming device in the direct proximity of a network node may be theoretically possible; however, such a placement might be undesirable due to security considerations. In this case, the location considered would have a higher placement cost than would a safer location. If there are no preferences for device locations, then without the loss of generality,

$$c_j = 1, \quad j = 1, 2, \ldots, n.$$

Though we have removed the non-convex covering constraints, this formulation remains computationally difficult. Notice that ONC is formulated as a MULTIDIMENSIONAL KNAPSACK PROBLEM which is known to be $\mathcal{NP}$-hard in general [1].

3. CONNECTIVITY FORMULATION

In the general WNJP, it is important that the distinction be made that the objective is not simply to jam some of the nodes, but to destroy the functionality of the underlying communication network. In this section, we use tools from graph theory to develop a method for suppressing the network by jamming those nodes with several communication links and derive an alternative formulation of the WNJP.
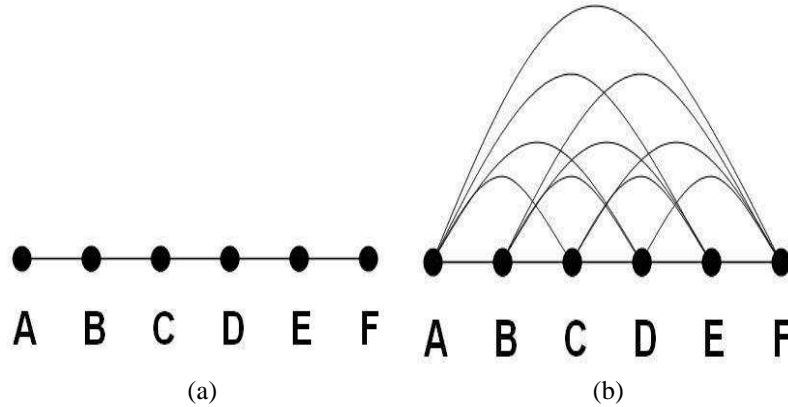

Figure 1: (a) Original graph G. (b) Transitive closure of G.

Given a graph $G = (V, E)$, the *transitive closure* of G is a graph $G' = (V, E')$, where $(i, j) \in E'$ if and only if there exists a path from i to j in G. Figure 1 provides an example of a graph and its transitive closure. Also, the *connectivity index* of a node is defined as the number of nodes reachable from that vertex (see Figure 2 for examples). To constrain the network connectivity in optimization models, we can impose constraints on the connectivity indices instead of using covering constraints.

We can now develop a formulation for the WNJP based on the connectivity index of the communication graph. We assume that the set of communication nodes $\mathcal{M} = \{1, 2, \ldots, m\}$ to be jammed is known and a set of possible locations $\mathcal{N} = \{1, 2, \ldots, n\}$ for the jamming devices is given. Let $S_i = \sum_{j=1}^{n} d_{ij} x_j$ denote the cumulative level of jamming at node i. Then node i is said to be jammed if S_i exceeds some threshold value C_i. We say that communication is severed between nodes i and j if at least one of the nodes is jammed. Further, let $y : V \times V \rightarrow \{0, 1\}$ be a surjection where $y_{ij} = 1$ if there exists a path from
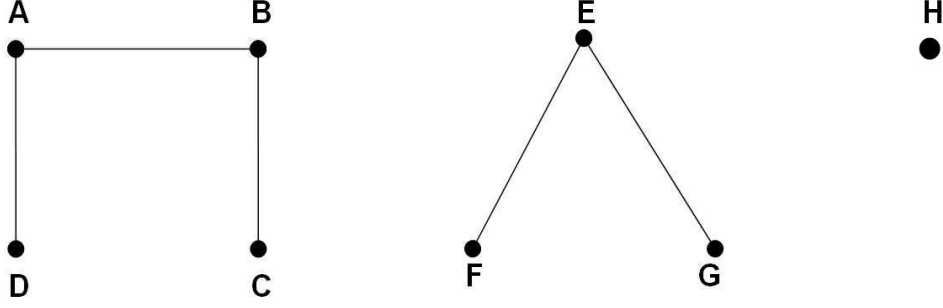
Figure 2: Connectivity Index of nodes A,B,C,D is 3. Connectivity Index of E,F,G is 2. Connectivity Index of H is 0.

node i to node j in the jammed network. Lastly, let $z : V \to \{0, 1\}$ where z_i returns 1 if node i is not jammed.

The objective of the CONNECTIVITY INDEX PROBLEM (CIP) formulation of the WNJP is to minimize total jamming cost subject to a constraint that the connectivity index of each node does not exceed some pre-described level L. The corresponding optimization problem is given as:

(CIP)   Minimize $$\sum_{j=1}^{n} c_j x_j \tag{7}$$

   s.t.

$$\sum_{j \neq i} y_{ij} \leq L, \ \forall \, i, j \in \mathcal{M} \tag{8}$$

$$M(1 - z_i) \geq S_i - C_i \geq -M z_i, \ \forall \, i \in \mathcal{M} \tag{9}$$

$$x_j \in \{0, 1\}, \ \forall \, j \in \mathcal{N} \tag{10}$$

$$z_i \in \{0, 1\} \ \forall \, i \in \mathcal{M}, \tag{11}$$

$$\forall \, i, j \in \mathcal{M}, y_{ij} = \begin{cases} 1, & \text{if } i \text{ reachable from } j \text{ in the jammed network} \\ 0, & \text{otherwise,} \end{cases} \tag{12}$$

where $M \in \mathbb{R}$ is some large constant.

Let $v : V \times V \to \{0, 1\}$ and $v' : V \times V \to \{0, 1\}$ be defined as follows:

$$v_{ij} = \begin{cases} 1, & \text{if } (i, j) \in E, \\ 0, & \text{otherwise,} \end{cases} \tag{13}$$

and

$$v'_{ij} = \begin{cases} 1, & \text{if } (i, j) \text{ exists in the jammed network,} \\ 0, & \text{otherwise.} \end{cases} \tag{14}$$

With this, we can formulate an equivalent integer program as

$$\textbf{(CIP-1)} \quad \text{Minimize} \quad \sum_{j=1}^{n} c_j x_j, \tag{15}$$

s.t.

$$y_{ij} \geq v'_{ij}, \ \forall \, i,j \in \mathcal{M}, \tag{16}$$

$$y_{ij} \geq y_{ik} y_{kj}, \ k \neq i,j; \ \forall \, i,j \in \mathcal{M}, \tag{17}$$

$$v'_{ij} \geq v_{ij} z_j z_i, \ i \neq j; \ \forall \, i,j \in \mathcal{M}, \tag{18}$$

$$\sum_{j=1}^{m} y_{ij} \leq L, \ j \neq i, \ \forall \, i \in \mathcal{M}, \tag{19}$$

$$M(1 - z_i) \ \geq \ S_i - C_i \geq -Mz_i, \ \forall \, i \in \mathcal{M}, \tag{20}$$

$$z_i \in \{0,1\}, \ \forall \, i \in \mathcal{M}, \tag{21}$$

$$x_j \in \{0,1\}, \ \forall \, j \in \mathcal{N}, \qquad y_{ij} \in \{0,1\} \ \forall \, i,j \in \mathcal{M}, \tag{22}$$

$$v_{ij} \in \{0,1\}, \ \forall \, i,j \in \mathcal{M}, \qquad v'_{ij} \in \{0,1\}, \ \forall \, i,j \in \mathcal{M}. \tag{23}$$

Lemma 1. *If* CIP *has an optimal solution then,* CIP-1 *has an optimal solution. Further, any optimal solution x^* of the optimization problem* CIP-1 *is an optimal solution of* CIP.

Proof. It is easy to establish that if i and j are reachable from each other in the jammed network then in CIP-1, $y_{ij} = 1$. Indeed, if i and j are adjacent then there exists a sequence of pairwise adjacent vertices:

$$\{(i_0, i_1), ..., (i_{m-1}, i_m)\}, \tag{24}$$

where $i_0 = i$, and $i_m = j$. Using induction it can be shown that $y_{i_0 i_k} = 1, \ \forall \, k = 1, 2, \ldots, m$. From (16), we have that $y_{i_k i_{k+1}} = 1$. If $y_{i_0 i_k} = 1$, then by (17), $y_{i_0 i_{k+1}} \geq y_{i_0 i_k} y_{i_k i_{k+1}} = 1$, which proves the induction step.

The proven property implies that in CIP-1:

$$\sum_{j \neq i} y_{ij} \geq \text{connectivity index of } i. \tag{25}$$

Therefore, if (x^*, y^*) and (x^{**}, y^{**}) are optimal solutions of CIP-1 and CIP correspondingly, then:

$$V(x^*) \geq V(x^{**}), \tag{26}$$

where V is the objective in CIP-1 and CIP.
As (x^{**}, y^{**}) is feasible in CIP, it can be easily checked that y^{**} satisfies all feasibility constraints in CIP-1 (it follows from the definition of y_{ij} in CIP). So, (x^{**}, y^{**}) is feasible in CIP-1; thus proving the first statement of the lemma.
Hence from CIP-1,

$$V(x^{**}) \geq V(x^*). \tag{27}$$

From (26) and (27):

$$V(x^{**}) = V(x^*). \tag{28}$$

Let us define y such that

$$y_{ij} = 1 \Leftrightarrow j \text{ is reachable from } i \text{ in the network jammed by } x^*.$$

Using (25), (x^*, y) is feasible in CIP-1, and hence optimal. From the construction of y it follows that (x^*, y) is feasible in CIP. Relying on (28) we can claim that x^* is an optimal solution of CIP. The lemma is proved. $\square$

We have therefore established a one-to-one correspondence between formulations CIP and CIP-1. Now, we can linearize the integer program CIP-1 by applying some standard transformations. The resulting linear 0-1 program, CIP-2 is given as

$$\textbf{(CIP-2)} \quad \text{Minimize} \quad \sum_{j=1}^{n} c_j x_j \tag{29}$$

s.t.

$$y_{ij} \geq v'_{ij}, \ \forall \, i, j = 1, \ldots, \mathcal{M}, \tag{30}$$

$$y_{ij} \geq y_{ik} + y_{kj} - 1, \ k \neq i, j; \ \forall \, i, j \in \mathcal{M}, \tag{31}$$

$$v'_{ij} \geq v_{ij} + z_j + z_i - 2, \ i \neq j; \ \forall \, i, j \in \mathcal{M}, \tag{32}$$

$$\sum_{j=1}^{I} y_{ij} \leq L, \ j \neq i, \ \forall \, i \in \mathcal{M}, \tag{33}$$

$$M(1 - z_i) \geq S_i - C_i \geq -M z_i, \ \forall \, i \in \mathcal{M}, \tag{34}$$

$$z_i \in \{0, 1\}, \ \forall \, i \in \mathcal{M}, \tag{35}$$

$$x_j \in \{0, 1\}, \ \forall \, j \in \mathcal{N}, \qquad y_{ij} \in \{0, 1\} \ \forall \, i, j \in \mathcal{M}, \tag{36}$$

$$v_{ij} \in \{0, 1\}, \ \forall \, i, j \in \mathcal{M}, \qquad v'_{ij} \in \{0, 1\}, \ \forall \, i, j \in \mathcal{M}. \tag{37}$$

In the following lemma, we provide a proof of equivalence between CIP-1 and CIP-2.

Lemma 2. *If* CIP-1 *has an optimal solution then* CIP-2 *has an optimal solution. Furthermore, any optimal solution* x^* *of* CIP-2 *is an optimal solution of* CIP-1.

Proof. For 0-1 variables the following equivalence holds:

$$y_{ij} \geq y_{ik} y_{kj} \Leftrightarrow y_{ij} \geq y_{ik} + y_{kj} - 1$$

The only differences between CIP-1 and CIP-2 are the constraints:

$$v'_{ij} = v_{ij} z_j z_i \tag{38}$$

$$v'_{ij} \geq v_{ij} + z_i + z_j - 2 \tag{39}$$

Note that (38) implies (39) $(v_{ij} z_j z_i \geq v_{ij} + z_i + z_j - 2)$. Therefore, the feasibility region of CIP-2 includes the feasibility region of CIP-1. This proves the first statement of the lemma.

From the last property we can also deduce that for all x_1, x_2 such that x_1 is an optimal solution of CIP-1, and x_2 is optimal for CIP-2, that

$$V(x_1) \geq V(x_2), \tag{40}$$

where $V(x)$ is the objective of CIP-1 and CIP-2.

Let (x^*, y^*, v'^*, z^*) be an optimal solution of CIP-2. Construct v''^* using the following rules:

$$v''_{ij}{}^* = \begin{cases} 1, & \text{if } v_{ij} + z_i^* + z_j^* - 2 = 1, \\ 0, & \text{otherwise.} \end{cases} \tag{41}$$

$v'_{ij}{}^* \geq v''_{ij}{}^* \Rightarrow (x^*, y^*, v''^*, z^*)$ is feasible in CIP-2 $(y_{ij} \geq v''_{ij}{}^*)$, hence optimal (the objective value is $V(x^*)$, which is optimal). Using (41), (v''^*, z^*) satisfies:

$$v''_{ij}{}^* = v_{ij} z_j^* z_i^*.$$

Using this we have that (x^*, y^*, v''^*, z^*) is feasible for CIP-1. If x_1 is an optimal solution of CIP-1 then:

$$V(x_1) \leq V(x^*) \tag{42}$$

On the other hand, using (40):

$$V(x^*) \leq V(x_1). \tag{43}$$

(42) and (43) together imply $V(x_1) = V(x^*)$. The last equality proves that x^* is an optimal solution of CIP-1. Thus, the lemma is proved. □

We have as a result of the above lemmata the following theorem which states that the optimal solution to the linearized integer program CIP-2 is an optimal solution to the original connectivity index problem CIP.

Theorem 1. *If* CIP *has an optimal solution then* CIP-2 *has an optimal solution. Furthermore, any optimal solution of* CIP-2 *is an optimal solution of* CIP.

Proof. The theorem is an immediate corollary of **Lemma 1** and **Lemma 2**. □

4. DETERMINISTIC SETUP WITH PERCENTILE CONSTRAINTS

As mentioned in Section 1, to suppress communication on a wireless network does not necessarily imply that all nodes must be jammed. It may be sufficient to jam some percentage of the total number of nodes in order to acquire an effective control over the network. Therefore we formulate the WNJP with percentile constraints which require that some percentage $\alpha \in [0, 1]$, of the nodes be jammed. This type of constraint is known as a Value at Risk (VaR) percentile constraint [4].

To incorporate VaR constraints into the ONC and ONC-1 formulations we can easily take advantage of the fact that both formulations are discrete 0-1 programming problems. Let $y : V \rightarrow \{0, 1\}$ where

$$y_i = \begin{cases} 1, & \text{if node } i \text{ is covered,} \\ 0, & \text{otherwise.} \end{cases} \tag{44}$$

Then to find the minimum number of locations of jamming devices that will allow for covering $\alpha \cdot 100\%$ of the network nodes with prescribed levels of jamming C_i, we must solve the following integer program

$$\textbf{(ONC-VaR)} \quad \text{Minimize} \quad \sum_{j=1}^{n} c_j x_j \tag{45}$$

$$\text{s.t.}$$

$$\sum_{i=1}^{m} y_i \geq \alpha m, \ i = 1, 2, \ldots, m, \tag{46}$$

$$\sum_{j=1}^{J} d_{ij} x_j \geq C_i y_i, \quad i = 1, 2, \ldots, m, \tag{47}$$

$$x_j \in \{0, 1\}, \quad j = 1, 2, \ldots, n, \tag{48}$$

$$y_i \in \{0, 1\}, \quad i = 1, 2, \ldots, m. \tag{49}$$

Notice that the only difference between this formulation and the ONC formulation is the addition of the m VaR constraints in (46) which ensure that the minimum required percentage of the nodes are jammed. The constraints in (47) enforce the coverage requirement C_i for each node i that is covered.

The approach is quite useful when the network structure is known entirely, because the constraints in ONC-VaR do not guarantee any level of coverage for the nodes with $y_i = 0$. However, this does not make the problem any easier to solve because the VaR type percentile constraints add an additional m integer variables to the problem.

In the same manner, we can reformulate the CONNECTIVITY INDEX PROBLEM formulation to include VaR type constraints. Let $\rho : V \rightarrow \mathbb{Z}^+$ be a function such that ρ_i returns the connectivity index of node i. That is, $\rho_i = \sum_{j=1, j\neq i}^{m} y_{ij}$. Further let $w : V \rightarrow \{0, 1\}$

be defined as

$$w_i \quad = \quad \begin{cases} 1, \text{ if } \rho_i \leq L, \\ 0, \text{ otherwise.} \end{cases} \tag{50}$$

With this, the connectivity index formulation of WNJP with VaR percentile constraints is given as

$$\textbf{(CIP-VaR)} \quad \text{Minimize} \quad \sum_{j=1}^{n} c_j x_j \tag{51}$$

$$\text{s.t.}$$

$$p_i \leq Lw_i + (1 - w_i)M, \ i = 1, 2, \ldots, m, \tag{52}$$

$$\sum_{i=1}^{m} w_i \geq \alpha m, \tag{53}$$

$$x_j \in \{0, 1\}, \ j = 1, 2, \ldots, n \tag{54}$$

$$w_i \in \{0, 1\}, \ i = 1, 2, \ldots, m, \tag{55}$$

$$p_i \in \{0, 1\}, \ i = 1, 2, \ldots, m, \tag{56}$$

where M is some large constant.

As with the ONC-VaR formulation, there are two drawbacks of CIP-VaR. First, there is no control guarantee at all on any of the remaining $(1 - \alpha) \cdot 100\%$ nodes. Secondly, the addition of a m binary variables adds a tremendous computational burden to the problem.

A more tractable approach is to impose a percentile constraint ensuring an average level of coverage $C_{\min}$ for $(1 - \alpha) \cdot 100\%$ of the worst (least) jammed nodes. This type of constraint can be formulated using the concept of Conditional Value-at-Risk (CVaR) [7, 8]. Developed by Rockafellar and Uryasev, CVaR is formally defined as a percentile risk measure constructed for estimation and control of risks in stochastic and uncertain environments. However, CVaR-based optimization techniques can also be applied in a deterministic percentile framework. For a description of CVaR methodology and related optimization techniques, the reader is referred to [7, 8].

Here, we present a formulation of the OPTIMAL NETWORK COVERING problem with CVaR-type percentile constraints resulting in the following mixed integer program:

$$\textbf{(ONC-CVaR)} \quad \text{Minimize} \sum_{j=1}^{n} c_j x_j \tag{57}$$

$$\text{subject to}$$

$$\zeta + \frac{1}{(1 - \alpha)I} \sum_{i=1}^{m} \max\left\{ C_{\min} - \sum_{j=1}^{n} x_j d_{ij} - \zeta, \ 0 \right\} \leq 0, \tag{58}$$

$$\zeta \in \mathbb{R}, \tag{59}$$

$$x_j \in \{0, 1\}. \tag{60}$$

The CVaR constraint (58) ensures that the average coverage across $(1 - \alpha) \cdot 100\%$ of the worst (least) covered nodes exceeds the minimal prescribed level $C_{\min}$. Consequently, the coverage of all other nodes in the network also exceeds $C_{\min}$.

The important point about this formulation is that we have not introduced additional integer variables to the problem in order to add the percentile constraints. Recall, that in ONC-VaR we introduced m discrete variables. Since we have to add only m real variables to replace max-expressions under the summation and a real variable ζ, this formulation is much easier to solve than ONC-VaR. In a similar manner, we can formulate the connectivity

index problem with the addition of CVaR constraints as follows:

$$\textbf{(CIP-CVaR)} \quad \text{Minimize} \sum_{j=1}^{n} c_j x_j \tag{61}$$

subject to

$$\zeta + \frac{1}{(1-\alpha)I} \sum_{i=1}^{m} \max\{\rho_i - L - \zeta,\, 0\} \leq 0, \tag{62}$$

$$\rho_i \in \mathbb{Z}, \tag{63}$$

$$\zeta \in \mathbb{R}. \tag{64}$$

Recall that ρ_i is the connectivity index of node i. Again, we see that in order to include the CVaR constraint, we only need to add $(m+1)$ real variables to the problem. Computationally, this will be much easier to solve than the CIP-VaR formulation as we will see in the next section.

5. CASE STUDIES

In order to demonstrate the advantages and disadvantages of the proposed formulations for the WNJP, we will present two case studies. The experiments were performed on a PC equipped with a 1.4MHz Intel Pentium® 4 processor with 1GB of RAM, working under the Microsoft Windows® XP SP1 operating system. In the first study, an example network is given and the problem is modeled using the proposed coverage formulation. The problem is then solved exactly using the commercial integer programming software package, CPLEX®. Next, we modify the problem to include VaR and CVaR constraints and again use CPLEX® to solve the resulting problems. Numerical results are presented and the three formulations are compared. In the second case study, we model and solve the problem using the connectivity index formulation. We then include percentile constraints re-optimize. Finally, we analyze the results.

Optimal Solutions	Regular Constraints	VaR Constraints
Number of Jammers	6	4
Level of Jamming	100% ∀ nodes	100% for 96% of nodes, 85% (of reqd.) for 4% of nodes
CPLEX® Time	0.81 sec	0.98 sec

Table 1: Optimal solutions using the coverage formulation with regular and VaR constraints.

5.1. **Coverage Formulation.** Here we present two networks and solve the WNJP using the network covering (ONC) formulation. The first network has 100 communication nodes and the number of available jamming devices is 36. The cost of placing a jamming device at location j, c_j is equal to 1 for all locations. This problem was solved using the regular constraints and the VaR type constraints. Recall that there is a set of possible locations at which jamming devices can be placed. In these examples, this set of points constitutes a uniform grid over the battlespace. The placement of the jamming devices from each solution can be seen in Figure 3. The numerical results detailing the level of jamming for the network nodes is given in Table 1. Notice that the VaR solution called for 33% less jamming devices than the original problem while providing almost the same jamming quality.

In the second example, the network has 100 communication nodes and 72 available jammers. This problem was solved using the regular constraints as well as both types

Opt Solns	Reg (all)	VaR (.9 conf)	CVaR (.7 conf)
# Jammers	9	8	7
Jamming Level	100% ∀ nodes	100% for 90% of nodes, 72% for 10% of nodes	100% for 57% of nodes, 90% for 20% of nodes, 76% for 23% of nodes
CPLEX® Time	15 sec	15h 55min 11sec	41 sec

Table 2: Optimal solutions using the coverage formulation with regular and VaR, and CVaR constraints.

of percentile constraints. The resulting graph is shown in Figure 4. The corresponding numerical results are given in Table 2.

In this example, the VaR formulation requires 11% less jamming devices with almost the same quality as the formulation with the standard constraints. However, this formulation requires nearly 16 hours of computation time. The CVaR formulation gives a solution with a very good jamming quality and requires 22% less jamming devices than the standard formulation and 11% less devices than the VaR formulation. Furthermore, the CVaR formulation requires an order of magnitude less computing time than the formulation with VaR constraints.

5.2. **Connectivity Formulation.** We now present a case study where the WNJP was solved using the connectivity index formulation (CIP). The communication graph consists of 30
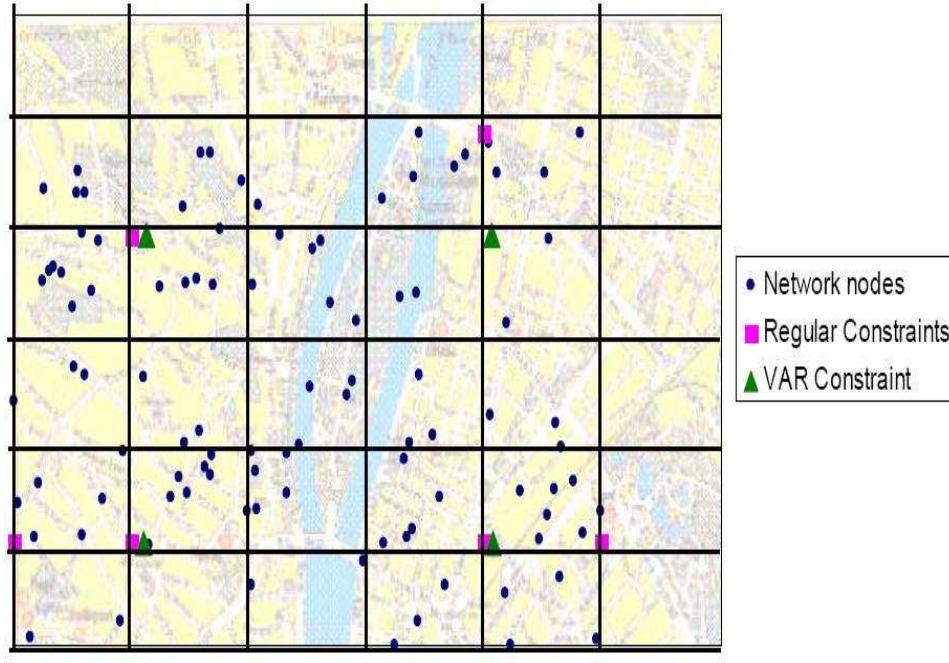

Figure 3: Case study 1. The placement of jammers is shown when the problem is solved using the original and VaR constraints.

nodes and 60 edges. The maximal number of jamming devices available is 36. We set the maximal allowed connectivity index of any node to be 3. In Figure 5 we can see the original graph with the communication links prior to jamming. The result of the VaR and CVaR solutions is seen in Figure 6. The confidence level for both the VaR and CVaR formulations was 0.9. Both formulations provide optimal solutions for the given instance. The resulting computation time for the VaR formulation was 15 minutes 34 seconds, while the CVaR formulation required only 7 minutes 33 seconds.

6. EXTENSIONS AND CONCLUSIONS

In this paper we introduced the deterministic WIRELESS NETWORK JAMMING PROBLEM and provided several formulations using node covering constraints as well as constraints on the connectivity indices of the network nodes. We also incorporated percentile constraints into the derived formulations. Further, we provided two case studies comparing the two formulations with and without the risk constraints.

With the introduction of this problem, we also recognize that several extensions can be made. For example, all of the formulations presented in this paper assume that the network topology of the enemy network is known. It is reasonable to assume that this is not always the case. In fact, there may be little or no prior information about the network to be jammed. In this case, stochastic formulations should be considered and analyzed.

A generalization of the node coverage formulation including uncertainties in the number of communication nodes and their coordinates might be considered. For the connectivity index problem, there might exist uncertainties in the number of network nodes, their locations, and the probability that a node will recover a jammed link. Also, efficient heuristics such as Greedy Randomized Adaptive Search Procedure (GRASP) [5], Genetic Algorithms [3], and Tabu Search [2], should be designed so that larger real-world instances can be
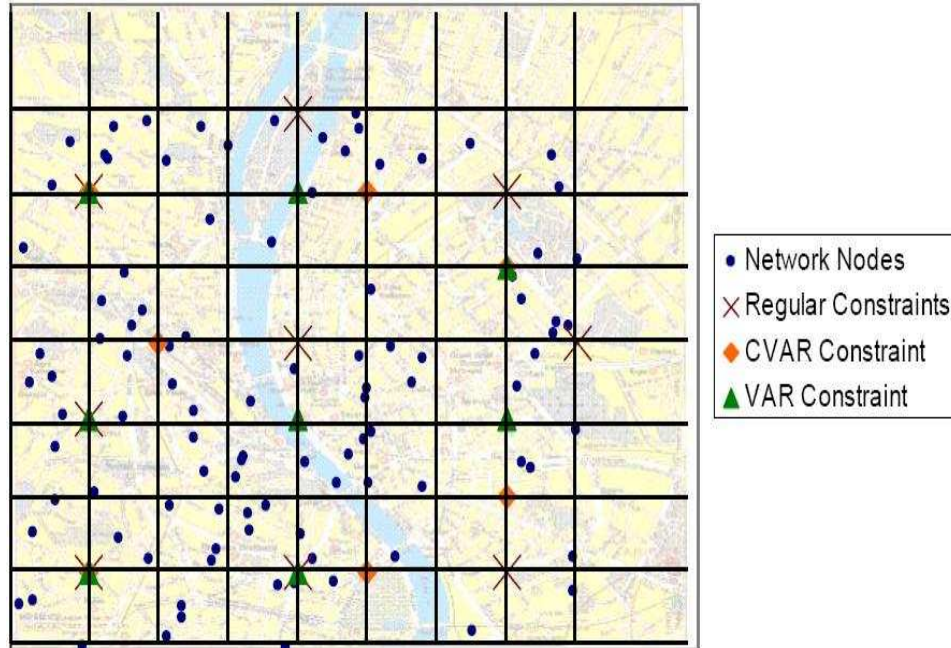

Figure 4: Case study 1 continued. The placement of jammers is shown when the problem is solved using VaR and CVaR constraints.
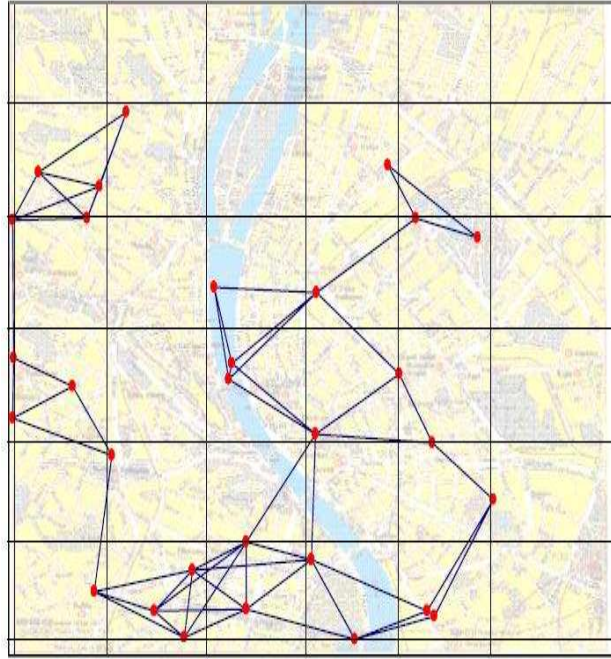
Figure 5: Case Study 2: Original graph.

solved. These are only a few ideas and extensions that can be derived from this new and interesting combinatorial optimization problem.
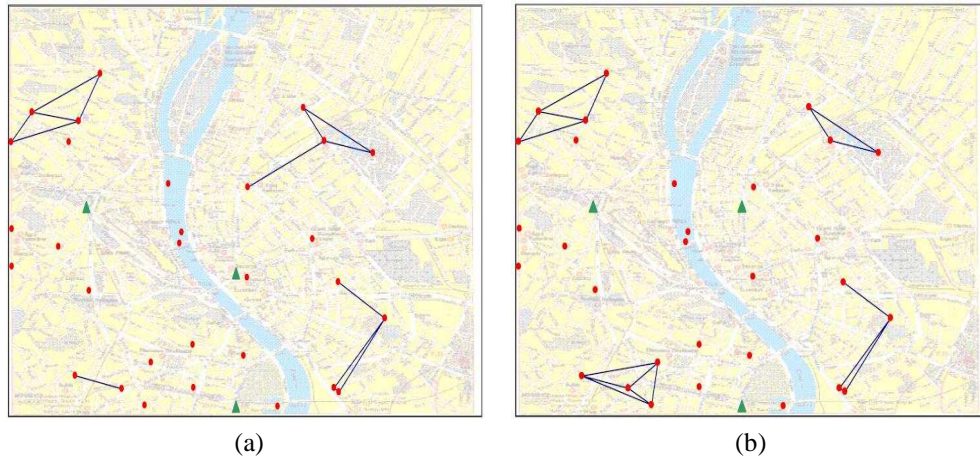

Figure 6: (a) VaR Solution. (b) CVaR Solution. In both cases, the triangles represent the jammer locations.

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